Task-Specific Grasp Selection for Underactuated Hands

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Abstract— In this paper, we propose an optimization scheme for deriving task-specific force closure grasps for underactuated robot hands. Motivated by recent neuroscientific studies on the human grasping behavior, a novel grasp strategy is built upon past analysis regarding the task-specificity of human grasps, that also complies with the recent soft synergy model of underactuated hands. Our scheme determines an efficient force closure grasp (i.e., configuration and contact points/forces) with a posture compatible with the desired task, taking into consideration the mechanical and geometric limitations imposed by the design of the hand and the object shape. The efficiency of the algorithm is verified through simulated paradigms on a hypothetical underactuated hand with the kinematic model of the DLR/HIT II five fingered robot hand.

I. INTRODUCTION

During the past decades, the design of dexterous multifingered hands, with plenty degrees of freedom, has received increased attention in an effort to reproduce the functionality and dexterity of human hands. However, building fully actuated multifingered hands requires mounting motors at all DOFs (degrees of freedom), which increases the design complexity, the weight and the cost of the hand. Therefore, in order to relax the aforementioned issues, researchers have focused on the design of underactuated hand mechanisms (i.e., hands with fewer actuators than DOFs), in which the kinematics are defined by coordinated patterns of joint movements. Such a direction was also supported by recent neuroscientific studies on human grasping. A series of experiments in [1], [2] verified that when humans grasp simple everyday life objects, aiming at performing simple manipulation tasks, the hand kinematics follow specific motion coordination patterns, commonly referred to as synergies. As it was shown in [1], after performing Principal Components Analysis (PCA) in kinematic data captured during human grasps, a high correlation between the hand's DOFs is evident. In particular, the first two principal components can account in most cases for more than 80% of the variance.

The aforementioned inference was the motivation behind the extensive research that emerged, not only in theory but also in a software and hardware level, emphasizing particularly on the grasping problem of underactuated hands. In [3], an optimization scheme for deriving form closure grasps by searching in a subspace of the hand's configuration space is proposed and numerical results are presented for real robot hand models. The subspace consists of a subset of all synergies that account for the generation of the grasping action. In [4], the property of form-closure is extended to apply for underactuated hands. In [5], a model is proposed for the generation of grasping forces by synergistic underactuated hands. In particular, the concept of *soft synergies* is introduced in order to take into account the compliance of the object and the hand mechanism. This model is also adopted in [6], where an optimization scheme is employed to calculate force closure grasps. In parallel, significant developments have been realized towards the hardware implementation of underactuated hands. In this field, the works presented in [7], [8], [9] and [10] are among the most noteworthy.

In the context of the ongoing research concerning underactuated hands, emphasis has been devoted to their association with the grasping tasks to be executed. In [11], the authors present a kinematic modeling for underactuated elastic hands, such that the classical manipulability measures can be applied to assess the hand's capability to execute a manipulation task. In [12], a manipulability analysis for soft synergy underactuated hands is implemented, aiming at evaluating their performance in the task space. Such an approach is compatible with our everyday life experience as well. When humans grasp objects, they intuitively adapt their hand posture according to the object and the task they want to perform. This observation has been confirmed by several studies. In particular, in [2] it was shown that when humans are able to estimate the weight and center of mass of an object, they tend to predetermine accordingly the grasping forces required for the object's lift and also select appropriate contact points that facilitate the task execution. In [13], the selection of grasping postures by humans was studied. More specifically, human subjects performed different manipulation tasks with various objects and post processing of their hands' kinematics verified that they actually adopted grasping postures which tended to maximize the force/velocity transmitted to the object along the direction required for the successful execution of the tasks considered.

The problem of deriving optimal grasps wrt the task to be executed has been studied in the past for fully actuated hands and many different approaches have been proposed (an overview of the most noteworthy among them can be found in [14]). The main difficulty of this problem arises from appropriately modeling a grasping task. In this direction, the idea of representing the wrench/twist transmission capabilities of a grasp with task ellipsoids has been widely adopted. Based on this representation, Li and Sastry, in

[15], first formulated a task-oriented quality measure, the maximization of which leads to a grasp that favors the execution of the task considered. They also formulated a corresponding optimization problem and provided numerical examples for a planar grasp of a two-fingered fully actuated hand.

In our paper, motivated by the aforementioned neuroscientific studies and based on the task representation using the task ellipsoids, we address the problem of deriving task-specific force closure grasps for underactuated hands. Given the recent advances in the field of synergistic hands, this problem needs to be studied through a new perspective, since the synergistic degrees of actuation (DoAs) may favor or complicate the execution of a desired task depending on the hand posture (configuration/contact points). Based on this observation and adopting the "soft synergy" hand model, we formulate an optimization scheme that leads to a force closure grasp of low contact force distribution with a posture (configuration/contact points) that favors the transmission of the required wrench/twist along the directions imposed by the task considered. Towards this goal, we use the transformation of the Hand-Object Jacobian [16], [17] in a metric similar to the "Task Compatibility Index", originally developed for manipulators in [18] and modified for grasping in [16]. Our scheme determines the configuration of the hand, along with its wrist's position/orientation and contact force distribution so that it is compatible with a given grasping task. The whole approach is independent of the specific degree of actuation which can simply be described through its corresponding "synergy" or "eigengrasp" matrices. Furthermore, the geometric and mechanical limitations, imposed by the hand design are also taken into consideration in the proposed formulation. Finally, it should be noted that although in this work we only consider precision grasps, the proposed scheme can be modified to derive power grasps as well.

The rest of the paper is organized as follows: Section II formulates the problem; Section III presents the methodology in details while Section IV verifies the efficiency of the proposed scheme through simulated paradigms on a hand with the kinematic model (DH parameters and DOFs) of the DLR/HIT Hand II. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

A. Rigid Body Grasping Model

Consider an n_c-fingered robot hand, consisting of n_q rotational DOFs in total, grasping an object with n_c fingertip contacts. Let us denote the contact wrench of the grasp by $\mathbf{f} = \begin{bmatrix} \mathbf{f}_1^T & ... & \mathbf{f}_{n_c}^T \end{bmatrix}^T \in \Re^{mn_c}$, where $\mathbf{f}_i \in \Re^m$ is the vector of the i-th generalized contact force, defined relative to a local frame $\{C_i\}$, with axes $\{n_i, t_i, o_i\}$, from which n_i is normal to the contact tangent plane and points towards the object, while the two others are orthogonal and lie on the contact tangent plane. The dimension m depends on the adopted contact model. In our analysis we adopt the *Hard Finger* (*HF*) model [19], which assumes that only the three *force* components of each contact wrench can be transmitted from each finger to the object. Thus, $m = 3$ in our case.

The contribution $\mathbf{g} \in \Re^6$ of the contact force distribution to the wrench applied at the object's center of mass, defined relative to a global reference frame $\{N\}$ is given by $\mathbf{g} = \mathbf{Gf}$ where $\mathbf{G} \in \Re^{6 \times mn_c}$ denotes the corresponding grasp matrix. The contact forces can be related to the joint torques $\boldsymbol{\tau} \in \Re^{n_q}$ through the Hand Jacobian $\mathbf{J} \in \Re^{mn_c \times n_q}$:

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{f} \tag{1}$$

Regarding the twist applied at the object, only the translational twist components of each contact are considered to contribute to it. In particular, let us denote the contact twist by the vector $\boldsymbol{\nu} = \begin{bmatrix} \boldsymbol{\nu}_1^T & ... & \boldsymbol{\nu}_{n_c}^T \end{bmatrix}^T \in \Re^{mn_c}$, where $\boldsymbol{\nu}_i \in \Re^m$ is the vector containing the translational velocity transmitted at the i-th contact, defined relative to $\{C_i\}$. Therefore, the object twist $\boldsymbol{\upsilon} \in \Re^6$ can be related to the contact twists as:

$$\boldsymbol{\nu} = \mathbf{G}^T \boldsymbol{\upsilon} \tag{2}$$

Finally, the contact twists can be related to the joint angular velocities $\dot{\mathbf{q}} \in \Re^{n_q}$ through the Hand Jacobian as follows:

$$\boldsymbol{\nu} = \mathbf{J}\dot{\mathbf{q}} \tag{3}$$

B. Introduction of Synergies

We introduce the concept of synergies in our grasp analysis as presented in the works of Santello et al. [1], [5] and Allen et al. [3]. We assume that the *full synergy* or *eigengrasp* matrix $\bar{\mathbf{S}} \in \Re^{n_q \times n_q}$ is available. $\bar{\mathbf{S}}$ consists of columns containing the principal components of the human grasp, ranked in a decreasing order wrt their relative contribution to the variance of the human grasp. Therefore, depending on the specific underactuated robot hand design (number of synergistic degrees of freedom n_s), a submatrix $\mathbf{S} \in \Re^{n_q \times n_s}$ ($1 \leq n_s \leq n_q$) can be extracted from $\bar{\mathbf{S}}$ in order to relate the high dimensional space of the hand's DOFs $\mathbf{q}_r \in \Re^{n_q}$ with the low dimensional space of the synergistic DoAs $\boldsymbol{\sigma} \in \Re^{n_s}$:

$$\mathbf{q}_r = \mathbf{S}\boldsymbol{\sigma} + \bar{\mathbf{q}} \tag{4}$$

where $\bar{\mathbf{q}} \in \Re^{n_q}$ denotes the mean values of the captured data used to extract the eigengrasp matrix $\bar{\mathbf{S}}$. Differentiating (4) wrt time and substituting into (3), we obtain:

$$\boldsymbol{\nu} = \mathbf{J}_s \dot{\boldsymbol{\sigma}} \tag{5}$$

where

$$\mathbf{J}_s = \mathbf{J}(\boldsymbol{\sigma})\mathbf{S} \tag{6}$$

Similarly, we may derive the relationship between the generalized synergistic forces $\boldsymbol{\eta} \in \Re^{n_s}$ and the torques on the joints:

$$\boldsymbol{\eta} = \mathbf{S}^T \boldsymbol{\tau} \tag{7}$$

Finally, employing (1) and (6) in (7), we get:

$$\boldsymbol{\eta} = \mathbf{J}_s^T \mathbf{f}. \tag{8}$$

C. The Soft Synergy

For an underactuated multifingered hand, the kinematic model (4) is not sufficient to realistically describe a grasp. The interaction with a grasped object needs to be modeled and introduced in the analysis as well. This has recently been done with the "Soft Synergy", a modeling framework [5], [6] which allows the hand's kinematics to be driven by synergies in a way that the final posture comports with the geometry of the grasped object. This is accomplished by considering the structural compliance of the hand as well as the stiffness of the object.
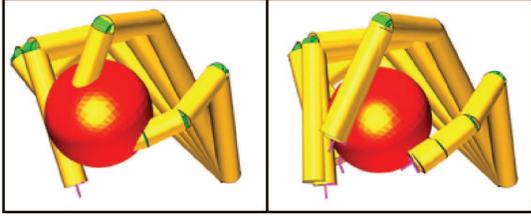
Fig. 1. Left: The reference configuration of the hand without contact interaction (i.e., $\mathbf{q}_r = \mathbf{S}\boldsymbol{\sigma} + \bar{\mathbf{q}}$). Right: The actual configuration of the hand with contact interaction and joint elasticity (i.e., $\mathbf{q} = \mathbf{q}_r - \mathbf{C}_q \delta\boldsymbol{\tau}$). This figure was extracted from [6].

More specifically, during a grasp, as the fingertips of the hand make contact with the object, forces appear and as a result, torques at the finger joints too. These torques modify the hand posture according to the contact and joint compliance so that the hand comports with the object shape (see Fig. 1). This phenomenon can be expressed through the following equation, relating each reference configuration $\mathbf{q}_r \in \mathfrak{R}^{n_q}$ to the actual hand configuration $\mathbf{q} \in \mathfrak{R}^{n_q}$:

$$\mathbf{q} = \mathbf{q}_r - \mathbf{C}_q \delta\boldsymbol{\tau} \qquad (9)$$

Eq. 9, via (4) yields:

$$\mathbf{q} = \mathbf{S}\boldsymbol{\sigma} + \bar{\mathbf{q}} - \mathbf{C}_q \delta\boldsymbol{\tau} \qquad (10)$$

where $\mathbf{C}_q \in \mathfrak{R}^{n_q \times n_q}$ is the diagonal joint compliance matrix which maps the torques on the joints due to fingertip contacts with the object to corresponding joint displacements.

D. Force Closure

Force closure [19] is the main prerequisite for stable grasps. It can be guaranteed by the satisfaction of two types conditions: *i)* the object's equilibrium and *ii)* the friction constraints. The balance equation for the generalized forces applied to the object, can be written as follows:

$$\mathbf{Gf} = -\mathbf{f}_{ext} \qquad (11)$$

where $\mathbf{f}_{ext} \in \mathfrak{R}^6$ is the external wrench applied at the object's center of mass. Adopting the grasping force decomposition model proposed in [6], the general solution to the force distribution problem (11) can be derived as follows:

$$\mathbf{f} = \mathbf{G}_K^R(-\mathbf{f}_{ext}) + (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})(\mathbf{K}_m\Delta\mathbf{p} + \mathbf{K}_c\delta\mathbf{p}) \qquad (12)$$

The first component (particular solution) of (12) accounts for the compensation of the external wrench applied to the object, while the second (homogeneous solution) represents the active internal forces of the grasp. In this model, the internal forces are produced i) through virtual displacements $\Delta\mathbf{p} \in \mathfrak{R}^{mn_c}$ of the fingertips due to the modification of the hand posture according to the soft synergy model (eq. 10) and ii) through infinitesimal deformations $\delta\mathbf{p} \in \mathfrak{R}^{mn_c}$ of the object at the contact points due to the object stiffness. These displacements that parameterize the homogeneous solution can be related to the corresponding joint displacements as

$$\Delta\mathbf{p} = \mathbf{J}\Delta\mathbf{q} \qquad (13)$$
$$\delta\mathbf{p} = \mathbf{J}\delta\mathbf{q} \qquad (14)$$

and consequently to corresponding synergistic displacements $\Delta\boldsymbol{\sigma} \in \mathfrak{R}^{n_s}$ and $\delta\boldsymbol{\sigma} \in \mathfrak{R}^{n_s}$ as:

$$\Delta\mathbf{p} = \mathbf{J}_s\Delta\boldsymbol{\sigma} \qquad (15)$$
$$\delta\mathbf{p} = \mathbf{J}_s\delta\boldsymbol{\sigma} \qquad (16)$$

Therefore, (12) can be reformulated, incorporating the synergistic underactuated hand design, as follows:

$$\mathbf{f} = \mathbf{G}_K^R(-\mathbf{f}_{ext}) + (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})(\mathbf{K}_m\mathbf{J}_s\Delta\boldsymbol{\sigma} + \mathbf{K}_c\mathbf{J}_s\delta\boldsymbol{\sigma}) \qquad (17)$$

As for the constants involved in the previous equations, $\mathbf{K}_m \in \mathfrak{R}^{mn_c \times mn_c}$ is the stiffness matrix that represents the structural compliance of the hand mechanism and $\mathbf{K}_c \in \mathfrak{R}^{mn_c \times mn_c}$ is the diagonal matrix introducing the contact stiffness (object and fingertip). $\mathbf{G}_K^R = \mathbf{K}_m\mathbf{G}^T(\mathbf{G}\mathbf{K}_m\mathbf{G}^T)^{-1}$ denotes the $\mathbf{K}_m$-weighted right inverse of the grasp matrix that minimizes the potential energy $\frac{1}{2}\delta\mathbf{p}^T\mathbf{K}_m\delta\mathbf{p}$ [6]. The stiffness matrix $\mathbf{K}_m$ can be computed as $\mathbf{K}_m = (\mathbf{C}_s + \mathbf{J}\mathbf{C}_q\mathbf{J}^T)^{-1}$ [20], where $\mathbf{C}_s \in \mathfrak{R}^{n_c \times n_c}$ is the diagonal structural compliance matrix representing the flexibility of the links and fingerpads.

Regarding the friction constraints, the *HF* model imposes nonlinear inequalities of the form $\sqrt{\mathrm{f}_{t_i}^2 + \mathrm{f}_{o_i}^2} \leqslant \mu\mathrm{f}_{n_i}, i = 1...n_c$ where f_{n_i}, f_{t_i}, f_{o_i} represent the contact force components along axes n_i, t_i and o_i respectively and μ denotes the friction coefficient between the contact surfaces of the fingers and the object. These inequalities, which are commonly referred to as "friction cone" constraints due to their geometrical representation, constrain the normal components of the contact forces to be non-negative, which indicates that the fingers tend to squeeze the object.

E. Grasp Quality Measures

As it can be inferred from the previous description, force closure is quite a generic criterion. For a multifingered hand with multiple degrees of freedom, there might be an infinite number of force closure grasps. In this paper, we focus on producing grasps that are compatible with i) the hand's mechanical design, ii) the hand's synergistic type of actuation and iii) the specifications of the grasping task to be executed. The need for such an approach arises from the fact that the robot hand's kinematics, as well as its ability to exert the desired wrench or twist to the grasped object is constrained by its underactuated design. Therefore, the object should be grasped in a way that a low contact force distribution can guarantee stability and a satisfactory (wrt the task considered) wrench/twist can be transmitted to the grasped object. Such requirements can be addressed through the optimization of *Grasp Quality* measures [16]. In this paper, apart from force closure, two more criteria are considered: a criterion associated with the contact force minimization and a criterion which quantifies the compatibility of a grasp with the specifications of a certain task.

Regarding the contact force minimization, the norm of the normal contact force components has been adopted:

$$F(\mathbf{f}) = \sqrt{\sum_{i=1}^{n_c} \mathrm{f}_{n_i}^2} \qquad (18)$$

The minimization of this metric, through the satisfaction of the friction cone constraints imposed by the force closure property, leads to the overall minimization of the contact force distribution.

To address the problem of producing a grasp (configuration/contact points) that favors the desired task execution, it is important that we first provide an appropriate task definition. Several approaches have been proposed towards modeling a task [14]. In our case, a task is described through its requirements in wrench/twist transmission to the grasped object. To optimally execute a grasping/manipulation

task, the transmission of an object wrench and/or twist of specific range towards a specific direction is required. Since the grasping posture can affect the wrench/twist transmission to the object, our objective is to derive a grasp (configuration/contact points) that maximizes the ability of the hand to meet the demands of the task. Towards this goal, we are based on the *Hand-Object Jacobian* transformation [16], [17], which we modify appropriately to build a grasp compatibility metric for the case of underactuated hands with synergistic actuation. In particular, employing (2) and (5), we derive:

$$\mathbf{G}^T \boldsymbol{v} = \mathbf{J}_s \dot{\boldsymbol{\sigma}} \qquad (19)$$

from which we arrive at:

$$\boldsymbol{v} = (\mathbf{G}^\dagger)^T \mathbf{J}_s \dot{\boldsymbol{\sigma}} \qquad (20)$$

where $\mathbf{G}^\dagger = \mathbf{G}^T (\mathbf{G}\mathbf{G}^T)^{-1} \in \mathfrak{R}^{mn_c \times 6}$ denotes the generalized right inverse of the grasp matrix. Let us now define the *Hand-Object Jacobian* for an underactuated hand with synergistic DoAs:

$$\mathbf{H}_s = \mathbf{G}^\dagger \mathbf{J}_s \qquad (21)$$

In this respect, (20) becomes:

$$\boldsymbol{v} = \mathbf{H}_s \dot{\boldsymbol{\sigma}} \qquad (22)$$

Similarly, we derive the relationship between the generalized synergy forces and the wrench applied to the object as follows:

$$\boldsymbol{\eta} = \mathbf{H}_s^T \mathbf{g} \qquad (23)$$

Notice from (22) that a unit ball in the synergy velocity space (i.e., $\dot{\boldsymbol{\sigma}}^T \dot{\boldsymbol{\sigma}} = 1$) is mapped to its contribution to the object twist through the following *velocity ellipsoid*:

$$\boldsymbol{v}^T (\mathbf{H}_s \mathbf{H}_s^T)^{-1} \boldsymbol{v} = 1 \qquad (24)$$

In the same direction, we conclude from (23) that a unit ball in the generalized synergy force space (i.e., $\boldsymbol{\eta}^T \boldsymbol{\eta} = 1$) is mapped to its contribution to the object wrench through the *force ellipsoid*:

$$\mathbf{g}^T (\mathbf{H}_s \mathbf{H}_s^T) \mathbf{g} = 1 \qquad (25)$$

Therefore, we can derive the expressions of the corresponding force and velocity transmission ratios respectively as:

$$\alpha = [\mathbf{d}^T (\mathbf{H}_s \mathbf{H}_s^T) \mathbf{d}]^{-1/2} \qquad (26)$$

$$\beta = [\mathbf{d}^T (\mathbf{H}_s \mathbf{H}_s^T)^{-1} \mathbf{d}]^{-1/2} \qquad (27)$$

where $\mathbf{d} \in \mathfrak{R}^6$ denotes the unit vector that defines a desired wrench/twist transmission direction. These ratios, when maximized, lead to a maximum transmission of twist or wrench to the object towards the direction specified by $\mathbf{d}$. This is equivalent to aligning the major axis of the corresponding ellipsoid to the direction defined by the task specifications. In particular, if we want to maximize the wrench transmission along the directions defined by the vectors $\mathbf{d}_i$, $i = 1, \cdots, k$, we need to maximize the corresponding transmission ratios α_i. Alternatively, in order to maximize the twist transmission along the directions defined by the vectors $\mathbf{d}_j$, $j = 1, \cdots, l$, we need to maximize the corresponding transmission ratios β_j. The aforementioned specifications can be achieved by minimizing the *Grasp Compatibility Index for Underactuated Hands with Synergistic Actuation*, inspired by [16], [18]:

$$C_\sigma = \sum_{i=1}^{k} w_i \alpha_i^{-2} + \sum_{j=1}^{l} w_j \beta_j^{-2} \qquad (28)$$

where w_i and w_j denote the weighting factors that represent

the relationship between the various wrench/twist transmission requirements.

III. THE TASK-SPECIFIC GRASP SELECTION SCHEME FOR UNDERACTUATED HANDS

In this paper, the problem of deriving an optimal force closure grasp wrt the force distribution and the task specifications for underactuated hands is formulated as an optimization scheme. The specific hand design is introduced in the formulation through the use of the "synergy" matrix. As decision variables we consider the synergistic infinitesimal displacements $\delta\boldsymbol{\sigma} \in \mathfrak{R}^{n_s}$, which contribute to the generation of the internal forces, along with the hand's configuration in the synergy space $\boldsymbol{\sigma} \in \mathfrak{R}^{n_s}$ and the wrist's position/orientation $\mathbf{w} \in \mathfrak{R}^6$. The contact force distribution is derived through (17). Finally, we assume that a dexterous robot manipulator will be able to place the wrist in the specific position and orientation extracted by the optimization algorithm, which can be done for example in an anthropomorphic way following the approach proposed in [21].

In particular, let the vector of the decision variables be denoted by $\boldsymbol{x} = \begin{bmatrix} \delta\boldsymbol{\sigma}^T & \boldsymbol{\sigma}^T & \mathbf{w}^T \end{bmatrix}^T \in \mathfrak{R}^{2n_s+6}$ and the objective function by $z = w_F F(\boldsymbol{x}) + w_C C_\sigma(\boldsymbol{x})$, where w_F and w_C are weighting factors representing the significance of the components presented in (18) and (28) respectively.

Hence, the optimization problem is formulated as follows:

$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\arg\min} \ z \qquad (29)$$

s.t.

$$\sqrt{\mathbf{f}_{t_i}^2 + \mathbf{f}_{o_i}^2} \leqslant \mu \mathbf{f}_{n_i}, \quad i = 1...n_c \qquad (30)$$

$$\mathbf{q}_{min} \leq \mathbf{q}(\boldsymbol{\sigma}) \leq \mathbf{q}_{max} \qquad (31)$$

$$\mathbf{p}_c^i(\boldsymbol{\sigma}, \mathbf{w}) \in \partial \mathbf{O}, i = 1,...,n_c \qquad (32)$$

where $\mathbf{q}_{min}$, $\mathbf{q}_{max} \in \mathfrak{R}^{n_q}$ denote the lower and upper limits of the joint angular displacements respectively and $\mathbf{O}$ is the space occupied by the object. The final actual configuration is derived via (10) by substituting:

$$\delta\boldsymbol{\tau} = \mathbf{J}^T (\mathbf{I} - \mathbf{G}_K^R \mathbf{G})(\mathbf{K}_m \mathbf{J}_s \Delta\boldsymbol{\sigma} + \mathbf{K}_c \mathbf{J}_s \delta\boldsymbol{\sigma}) \qquad (33)$$

which represents the torques on the joints due to contact and joint compliance. The term $\Delta\boldsymbol{\sigma}$ represents the modification of the hand posture as forced by the object shape and compliance as described in [6]. Furthermore, the selection of $\boldsymbol{\sigma}$ and $\delta\boldsymbol{\sigma}$ guarantees not only that the fingertips make contact with the object, but also that the internal forces produced are such that the friction cone constraints are satisfied. Finally, the constraint (32) represents the requirement that the fingertip contact locations $\mathbf{p}_c^i \in \mathfrak{R}^3$, calculated by the forward kinematics for the final modified posture lie on the object's surface $\partial \mathbf{O}$.

Remark 1. In the case of multifingered hands with an abduction/adduction DOF at each finger base frame, the collision avoidance between the fingers opposed to the thumb can also be ensured. Such constraints can be expressed for example through inequalities of the form $q_0^i < q_0^{i+1}$, $i = 1,...,n_c - 1$ where $q_0^1,..., q_0^{n_c}$ denote the angular displacements of the abduction/adduction DOFs of all fingers opposed to the thumb, ranked in their physical order (e.g. index, middle, ring and pinky for a five-fingered hand).

Remark 2. An additional requirement that no link of the robot hand penetrates the grasped object may be introduced by a constraint of the form $\overline{(\mathbf{p}_i, \mathbf{p}_j)} \bigcap \mathbf{O} = \varnothing$ for all neighboring robot hand joints i, j, where the vectors $\mathbf{p}_i, \mathbf{p}_j \in \Re^3$, $i, j = 1, ..., n_q$ represent the locations of the corresponding joints. Given the analytical description of the object's geometry, such constraints can be easily formulated as inequality constraints that can be integrated in the optimization scheme.

IV. RESULTS

A hypothetical hand with the kinematic model (DH parameters, DOFs) of the DLR/HIT II robot hand [22] was considered for the simulated paradigms. DLR/HIT II is a multifingered robot hand with five kinematically identical fingers. Each finger has four DOFs, the last two of which are mechanically coupled with a transmission ratio 1:1.

The efficiency of the proposed optimization scheme is tested by considering simple grasping tasks with a cylindrical object of diameter 6 cm, height 15 cm and weight 200 gr. The friction coefficient between the surface of the fingers and the object was set to be 0.8, the diagonal elements of the stiffness matrix $\mathbf{K}_c$ were considered to be equal to 100.000 N/m, while the diagonal elements of $\mathbf{C}_s$ and $\mathbf{C}_q$ were set equal to 0.1 N/m and 0.1 Nm/° respectively. The grasping tasks were described as directions of desired wrench transmission to the object's center of mass. Regarding the derivation of the numerical results presented, the *SQP* algorithm [23] for constrained nonlinear optimization provided by the MATLAB Optimization Toolbox (Mathworks Inc.) was used. The algorithm was initialized at the same infeasible point (no contact with the object was made) in all cases. Due to the highly nonlinear nature of the problem, different initializations would lead to different local minima and different corresponding final postures. However, a significant advantage of the proposed optimization algorithm is its ability to converge to a desired solution after very few iterations, in contrast to global methods which require much higher computational cost.

In the subsequent simulations, we altered the number of synergistic DoAs n_s of the hand and considered two different grasping tasks. We also studied the effect of various weighting factors w_F and w_C. It should be noticed that the full synergy matrix which defines the synergistic DoAs was extracted from a Principal Component Analysis on human data collected in our laboratory. In particular, the hand kinematics of three different human subjects were captured using the Cyberglove II (Cyberglove Systems) motion capture system (dataglove) while they were performing simple grasping tasks with every day life objects.

TABLE I
THE EFFECT OF THE NUMBER OF DOAS

n_s	Translation along y axis			Translation along x axis		
	F (N)	angle (°)	No. of Iterations	F	angle (°)	No. of Iterations
1	26.06	63.62	42	27.01	67.85	46
2	22.09	56.27	54	14.89	48.83	103
3	15.04	13.95	72	12.96	17.22	119
15	1.97	3.43	192	2.36	1.77	197

Figs. 2 and 3 depict grasps, along with their corresponding force ellipsoids, generated with the proposed grasp selection

TABLE II
THE EFFECT OF THE WEIGHTS RATIO

Ratio w_F/w_C	Translation along green axis, 3 DOAs		
	F (N)	angle (°)	No. of Iterations
10^{-9}	67.30	7.86	97
10^{-6}	31.77	22.38	439
1	1.98	47.13	217

scheme. More specifically, in Fig. 2, a grasp is selected for performing a translation task towards the direction denoted by the cyan vector. The task was considered to be equivalent to force transmission along that axis. Results are provided for various DoAs, namely 1, 2, 3 and 15 (i.e., fully actuated) in subfigures 2.a, 2.b, 2.c and 2.d respectively. Notice that when the hand is actuated with very few DoAs, the grasp is far from being task compatible. However, increasing n_s leads to more optimized postures with respect to the task specificity metric C_σ, as the major axes of their force ellipsoids are more aligned with the desired direction (the angle between them is smaller). This can also be verified from the left side of Table I, where the angles between the task direction and the major axes of the force ellipsoid are presented. Moreover, as n_s increases, the force distribution keeps decreasing. At the same time however, the number of iterations needed until the convergence of the algorithm increases, which seems reasonable owing to the increase of the number of decision variables. Regarding the postures adopted, we can see that although the results obtained for 1, 2 and 3 DoAs are similar, the improvement of the grasp quality is significant. This comes from the fact that as n_s increases, the variety of achievable postures becomes wider. This is particularly evident in Fig. 2.d, where the 15 DoAs led a posture with a significantly improved grasp quality. When the hand's kinematics are dictated by 15 different DoAs (i.e., it is fully actuated), it can achieve more complex postures and consequently adapt better to the specifications of a given task. Finally, it should be mentioned that in all the aforementioned runs, the ratio w_F/w_C was set equal to 10^{-6}.

Similar comments can be made for Fig. 3, where the grasp is selected so that it facilitates the execution of a translation task along the direction indicated. The subfigures 3.a, 3.b, 3.c, 3.d depict the results for a hand with 1, 2, 3 and 15 DoAs respectively. As n_s increases, better alignment between the task direction and the major axis of the force ellipsoid is achieved. This is also confirmed in the right three columns of Table I where the corresponding angles are presented. Similarly with the previous task, the required force distribution decreases as n_s increases. Finally, in this case the ratio w_F/w_C was constantly set equal to 10^{-5}.

Table II contains quantitative results concerning the dependence of the algorithm on the selection of the weights of the objective function components F and C_σ. In order to compare the results for various weights, we considered the same task (translation along the green axis in Fig. 2) and DoAs ($n_s = 3$). We can see that when the ratio w_F/w_C is small, the alignment of the force ellipsoid's major axis with the desired direction is satisfactory (the corresponding angle is small) while less significance is attributed to the force distribution, which tends to be high. However, as the ratio
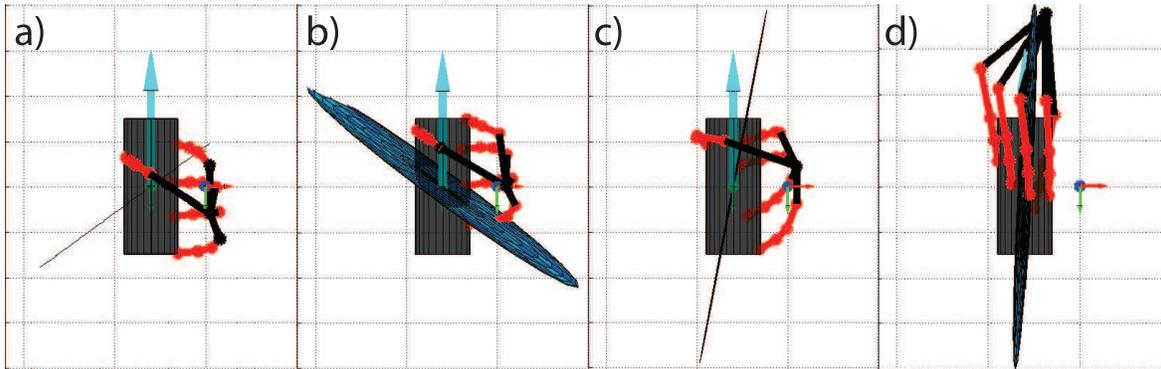
Fig. 2. Grasps of a cylindrical object aiming at translating the object towards the direction of the cyan vector. The subfigures 2.a, 2.b, 2.c, 2.d present simulation results produced with the proposed scheme for 1, 2, 3 and 15 DoAs respectively. The corresponding ellipsoids are drawn in blue.
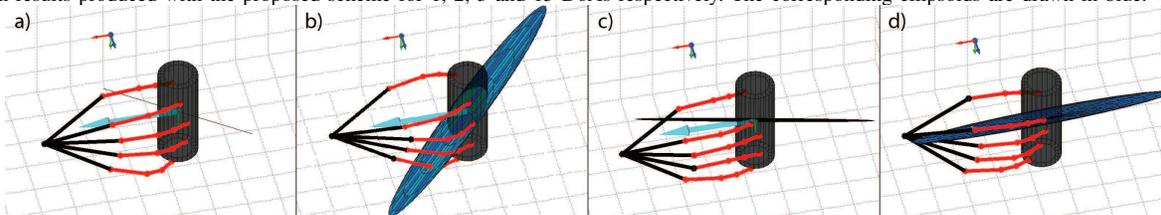

Fig. 3. Grasps of a cylindrical object aiming at pushing the object towards the direction denoted by the cyan vector. The subfigures 3.a, 3.b, 3.c, 3.d present simulation results produced with the proposed scheme for 1, 2, 3 and 15 DoAs respectively. The corresponding ellipsoids are drawn in blue.

increases, the force distribution is getting lower, causing the alignment to deteriorate.

V. Future Directions

Future research directions involve the integration of the proposed algorithm with a post-optimality procedure for improving grasp quality, as described in [24], aiming at reinforcing the robustness of the methodology against uncertainties regarding the object as well as the robotic hand artifact itself. Besides, we plan on performing experiments with real underactuated hands such as the one presented in [10] in order to verify the efficiency of the proposed grasping strategy.

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